cients K_I and K_{II} differ from zero; these cannot be determined separately by the method used. However, with an analysis of the conditions for the propagation of a crack, the value of the rate of evolution of elastic energy itself, which is determined directly in the given method, is important. The method can be applied to bodies of complex form with cracks also having a complex form.

LITERATURE CITED

1. G. R. Irwin, "Analysis of stresses and strains near the end of a crack traversing a plate," J. Appl. Mech., 24, No. 3, 361 (1957).
2. G. Sih, P. Paris, and F. Erdogan, "Crack-tip stress intensity factors for plane extension and plate bending problems," J. Appl. Mech., 29E, No. 2 (1962).
3. P. Paris and G. Sih, "Analysis of the stressed state around cracks," in: Applied Problems in Viscous Failure [Russian translation], Izd. Mir, Moscow (1968), pp. 64-142.
4. L. P. Frantsuzova, "Experimental investigation of the propagation of cracks in a model of a laminar plate," in: The Dynamics of a Continuous Medium, Nos. 19-20 [in Russian], Izd. Inst. Gidrodinam., Sibirsk. Otd. Akad. Nauk SSSR, Novosibirsk (1974), pp. 141-145.

SOLUTION IN THE FORM OF SERIES OF LEGENDRE POLYNOMIALS OF AN AXISYMMETRIC MIXED PROBLEM FOR A HOLLOW ELASTIC CYLINDER

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In this paper it is shown that the method proposed in [1] for the solution of a plane mixed problem of the theory of elasticity can be used also for the solution of an axisymmetric mixed problem for a hollow elastic cylinder.

1. Statement of the Problem

In the case of axisymmetric elastic deformation the equations of equilibrium and Hooke's law can be written in the form

$$\frac{\partial p}{\partial x}+\frac{\partial \tau}{\partial y}+\gamma_1=0,\quad \frac{\partial \tau}{\partial x}+\frac{\partial q}{\partial y}-t+\gamma_2=0,$$
$$p-r\left(\lambda\varepsilon+2\mu\frac{\partial u}{\partial x}\right)=0,\quad q-r\left(\lambda\varepsilon+2\mu\frac{\partial v}{\partial y}\right)=0,$$
$$\tau-\mu r\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)=0,\quad t-\lambda\varepsilon-2\mu\frac{v}{r}=0,$$

where

$$\varepsilon=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{v}{r},\quad \lambda=\frac{\nu E}{(1+\nu)(1-2\nu)},\quad 0\leqslant\nu<\frac{1}{2},\quad E,\mu>0,$$
$$p=r\sigma_z,\ q=r\sigma_r,\ \tau=r\sigma_{rz},\ t=\sigma_\varphi,\ u=u_z,\ v=u_r,$$
$$x=z-z_0,\ y=r-r_0;$$

r, φ, and z are the cylindrical coordinates; σ_r, σ_φ, σ_z, σ_{rz}, u_r, and u_z are the components of the stress tensor and the displacement vector in the cylindrical coordinate system; γ_1 and γ_2 are the mass forces; z_0 and r_0 are constants; E is Young's modulus; μ is the shear modulus; and ν is Poisson's ratio.

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We confine ourselves to the case where $|x| \leq 1$, $|y| \leq 1$, $r_0 > 1$, and by transformation of the sought functions the problem is reduced to the finding of the functions p, q, τ, t, u, and v, which satisfy the equations

$$\frac{\partial p}{\partial x} + \frac{\partial \tau}{\partial y} + f_1 = 0, \quad \frac{\partial \tau}{\partial x} + \frac{\partial q}{\partial y} - t + f_2 = 0,$$
$$p - r\left(\lambda\varepsilon + 2\mu \frac{\partial u}{\partial x}\right) + f_3 = 0, \quad q - r\left(\lambda\varepsilon + 2\mu \frac{\partial v}{\partial y}\right) + f_4 = 0,$$
$$\tau - \mu r\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + f_5 = 0, \quad t - \lambda\varepsilon - 2\mu \frac{v}{r} = 0,$$
$$\varepsilon = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{v}{r}$$

and the boundary conditions

$$(pu)_{x=\pm 1} = (qv)_{y=\pm 1} = (\tau v)_{x=\pm 1} = (\tau u)_{y=\pm 1} = 0, \tag{1.1}$$

where f_α, $\alpha = 1, \ldots, 5$ are known functions summable in square with respect to $\Omega = \{x, y \mid x \in [-1, 1], y \in [-1, 1]\}$. We assume that in each of Eqs. (1.1) one of the functions being multiplied is zero on the entire side of the square.

If in the case of the displacement of the cylinder as an absolutely rigid body we have the equation u = 0, then the conditions (1.1) will be supplemented by the equation

$$\int_\Omega u\, d\Omega = 0. \tag{1.2}$$

In this case the function f_1 cannot be arbitrary; it must satisfy the condition

$$\int_\Omega f_1\, d\Omega = 0.$$

2. Approximate Solution

We denote

$$\begin{aligned}
p^{nm} &= \sum_{k=0}^{n}\sum_{i=0}^{m} p_{ki}^{nm} P_k Q_i, & q^{nm} &= \sum_{k=0}^{n}\sum_{i=0}^{m} q_{ki}^{nm} P_k Q_i,\\
\tau_1^{nm} &= \sum_{k=0}^{n-1}\sum_{i=0}^{m+1} \tau_{ki}^{nm} P_k Q_i, & \tau_2^{nm} &= \sum_{k=0}^{n+1}\sum_{i=0}^{m-1} \tau_{ki}^{nm} P_k Q_i,\\
u_0^{nm} &= \sum_{k=0}^{n}\sum_{i=0}^{m-1} u_{ki}^{nm} P_k Q_i, & v_0^{nm} &= \sum_{k=0}^{n-1}\sum_{i=0}^{m} v_{ki}^{nm} P_k Q_i,\\
u_1^{nm} &= \sum_{k=0}^{n}\sum_{i=0}^{m+1} u_{ki}^{nm} P_k Q_i, & v_1^{nm} &= \sum_{k=0}^{n+1}\sum_{i=0}^{m} v_{ki}^{nm} P_k Q_i,\\
u_2^{nm} &= \sum_{k=0}^{n+2}\sum_{i=0}^{m-1} u_{ki}^{nm} P_k Q_i, & v_2^{nm} &= \sum_{k=0}^{n-1}\sum_{i=0}^{m+2} v_{ki}^{nm} P_k Q_i,\\
& & t^{nm} &= \sum_{k=0}^{n-1}\sum_{i=0}^{m} t_{ki}^{nm} P_k Q_i.
\end{aligned} \tag{2.1}$$

Here n, m ≥ 1; p_{ki}^{nm}, q_{ki}^{nm}, τ_{ki}^{nm}, u_{ki}^{nm}, v_{ki}^{nm}, and t_{ki}^{nm} are constants; $P_k = P_k(y)$ and $Q_i = Q_i(x)$ are Legendre polynominals that are orthogonal on the segment [−1, 1]; k and i are powers of the polynomials.

We stipulate that the functions (2.1) satisfy the equations

$$\int_\Omega \left(\frac{\partial p^{nm}}{\partial x} + \frac{\partial \tau_2^{nm}}{\partial y} + f_1\right) P_k Q_i\, d\Omega = 0, \quad k = 0, 1, \ldots, n, \ i = 0, 1, \ldots, m-1;$$

$$\int_\Omega \left(\frac{\partial \tau_1^{nm}}{\partial x} + \frac{\partial q^{nm}}{\partial y} - t^{nm} + f_2\right) P_k Q_i\, d\Omega = 0,$$
$$k = 0, 1, \ldots, n-1, \ i = 0, 1, \ldots, m;$$

$$\int_\Omega \left[p^{nm} - r\left(\lambda\varepsilon^{nm} + 2\mu \frac{\partial u_1^{nm}}{\partial x}\right) + f_3\right] P_k Q_i\, d\Omega = 0,$$

$$\int_{\Omega}\left[q^{nm}-r\left(\lambda\varepsilon^{nm}+2\mu\frac{\partial v_1^{nm}}{\partial y}\right)+f_4\right]P_kQ_i d\Omega=0,$$

$$k=0,1,\ldots,n,\ i=0,1,\ldots,m; \tag{2.2}$$

$$\int_{\Omega}\left[\tau_2^{nm}-\mu r\left(\frac{\partial u_2^{nm}}{\partial y}+\frac{\partial v_2^{nm}}{\partial x}\right)+f_5\right]P_kQ_i d\Omega=0,$$

$$k=0,1,\ldots,n+1,\ i=0,1,\ldots,m-1;$$

$$\int_{\Omega}\left[\tau_1^{nm}-\mu r\left(\frac{\partial u_2^{nm}}{\partial y}+\frac{\partial v_2^{nm}}{\partial x}\right)+f_5\right]P_kQ_i d\Omega=0,$$

$$k=0,1,\ldots,n-1,\ i=0,1,\ldots,m+1;$$

$$\int_{\Omega}\left(t^{nm}-\lambda\varepsilon^{nm}-2\mu\frac{v_0^{nm}}{r}\right)P_kQ_i d\Omega=0,$$

$$k=0,1,\ldots,n-1,\ i=0,1,\ldots,m,$$

$$\varepsilon^{nm}=\frac{\partial u_1^{nm}}{\partial x}+\frac{\partial v_1^{nm}}{\partial y}+\frac{v_0^{nm}}{r},\quad r=r_0+y,\quad r_0>1$$

and the zero boundary conditions

$$(p^{nm}u_1^{nm})_{x=\pm1}=(q^{nm}v_1^{nm})_{y=\pm1}=(\tau_1^{nm}v_2^{nm})_{x=\pm1}=(\tau_2^{nm}u_2^{nm})_{y=\pm1}=0. \tag{2.3}$$

It is assumed that in each of Eqs. (2.3) one of the factors [just as in (1.1)] is zero on the entire side of the square.

If the formulation of the problem contains Eq. (1.2), then in the system (2.2) the equation

$$\int_{\Omega}\left(\frac{\partial p^{nm}}{\partial x}+\frac{\partial \tau_2^{nm}}{\partial y}+f_1\right)d\Omega=0$$

is replaced by the equation $u_{00}^{nm}=0$.

Equations (2.2) and (2.3) form a closed system relative to the constants in the functions (2.1).

From (2.2), (2.3) we find

$$\int_{\Omega}\left[f_1u_0^{nm}+f_2v_0^{nm}+f_3\frac{\partial u_1^{nm}}{\partial x}+f_4\frac{\partial v_1^{nm}}{\partial y}+f_5\left(\frac{\partial u_2^{nm}}{\partial y}+\frac{\partial v_2^{nm}}{\partial x}\right)\right]d\Omega=\int_{\Omega}g_{nm}r d\Omega,$$

$$g_{nm}=\lambda(\varepsilon^{nm})^2+2\mu\left[\left(\frac{\partial u_1^{nm}}{\partial x}\right)^2+\left(\frac{\partial v_1^{nm}}{\partial y}\right)^2+\left(\frac{v_0^{nm}}{r}\right)^2+\frac{1}{2}\left(\frac{\partial u_2^{nm}}{\partial y}+\frac{\partial v_2^{nm}}{\partial x}\right)^2\right]. \tag{2.4}$$

Since $r\geq r_0-1$, we have

$$E_{nm}=\int_{\Omega}g_{nm}d\Omega\leq\frac{1}{r_0-1}\int_{\Omega}g_{nm}r d\Omega. \tag{2.5}$$

Using the inequalities (4.8), (4.10), and (4.11) of [1], we can prove that

$$\max\{\|u_1^{nm}\|,\|u_2^{nm}\|,\|v_1^{nm}\|,\|v_2^{nm}\|\}\leq CE_{nm}^{1/2}, \tag{2.6}$$

where the symbol $\|\ \|$ denotes the norm in $L_2(\Omega)$. By the letter C in (2.6) we have denoted a constant not depending on n and m. From (2.3) – (2.6) we find that the zero solution of the homogeneous system of equations (2.2), (2.3) is unique and, consequently, the determinant of this system is nonzero.

The functions (2.1), which satisfy the system (2.2), (2.3), are the approximate solution of the mixed problem for a hollow cylinder. The functions p^{nm}, q^{nm} are an approximation of the functions p and q. The functions τ_1^{nm}, τ_2^{nm} can be regarded as an approximation, multiplied by r, of the shear stresses on planes with normals directed along the z and r axes, respectively. The parity law of the shear stresses τ_1^{nm}, τ_2^{nm} is approximately satisfied by

$$\int_\Omega (\tau_1^{nm} - \tau_2^{nm}) P_k Q_i d\Omega = 0, \quad k = 0, 1, \ldots, n-1, \; i = 0, 1, \ldots, m-1.$$

In the role of an approximation of the function τ we can in fact consider the function

$$\tau^{nm} = \sum_{k=0}^{n-1} \sum_{i=0}^{m+1} \tau_{ki}^{nm} P_k Q_i + \sum_{k=n}^{n+1} \sum_{i=0}^{m-1} \tau_{ki}^{nm} P_k Q_i.$$

Obviously τ^{nm} satisfies Eqs. (2.2), if instead of τ_1^{nm}, τ_2^{nm} we substitute τ^{nm}, and it approximately satisfies, as is indicated in (2.3), the boundary conditions (1.1) for τ.

3. Convergence of Approximate Solutions

Functions p, q, τ, u, and v satisfying the equations

$$\begin{gathered}
\int_\Omega \left(\frac{\partial p}{\partial x} + \frac{\partial \tau}{\partial y} + f_1\right)\omega_1 d\Omega = 0, \quad \int_\Omega \left(\frac{\partial \tau}{\partial x} + \frac{\partial q}{\partial y} - t + f_2\right)\omega_2 d\Omega = 0, \\
\int_\Omega \left[q - r\left(\lambda\varepsilon + 2\mu\frac{\partial v}{\partial y}\right) + f_4\right]\omega_4 d\Omega = 0, \quad \int_\Omega \left[p - r\left(\lambda\varepsilon + 2\mu\frac{\partial u}{\partial x}\right) + f_3\right]\omega_3 d\Omega = 0, \\
\int_\Omega \left(t - \lambda\varepsilon - 2\mu\frac{v}{r}\right)\omega_6 d\Omega = 0, \quad \int_\Omega \left[\tau - \mu r\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + f_5\right]\omega_5 d\Omega = 0, \\
\int_\Omega \left(\frac{\partial p_*}{\partial x} u + p_* \frac{\partial u}{\partial x}\right) d\Omega = 0, \quad \int_\Omega \left(\frac{\partial q_*}{\partial y} v + q_* \frac{\partial v}{\partial y}\right) d\Omega = 0, \\
\int_\Omega \left[\frac{\partial \tau_*}{\partial y} u + \frac{\partial \tau_*}{\partial x} v + \tau_*\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right] d\Omega = 0, \\
\int_\Omega \left(p\frac{\partial u_*}{\partial x} + \tau\frac{\partial u_*}{\partial y} - f_1 u_*\right) d\Omega = 0, \quad \int_\Omega \left[\tau\frac{\partial v_*}{\partial x} + q\frac{\partial v_*}{\partial y} + (t - f_2) v_*\right] d\Omega = 0
\end{gathered} \tag{3.1}$$

and the inequality

$$\begin{gathered}
\int_\Omega \left[f_1 u + f_2 v + f_3 \frac{\partial u}{\partial x} + f_4 \frac{\partial v}{\partial y} + f_5\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right] d\Omega \geqslant \\
\geqslant \int_\Omega \left\{\lambda\varepsilon^2 + 2\mu\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{v}{r}\right)^2 + \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right]\right\} r d\Omega,
\end{gathered} \tag{3.2}$$

are called the generalized solution of the mixed problem for a hollow elastic cylinder. Here

$$\varepsilon = \partial u/\partial x + \partial v/\partial y + v/r,$$

ω_k, k=1, 2, . . ., 6 are arbitrary functions belonging to $L_2(\Omega)$; p_*, q_*, τ_*, u_*, v_* are arbitrary functions satisfying the conditions

$$\begin{gathered}
(p_* u_*)_{x=\pm 1} = (q_* v_*)_{y=\pm 1} = (\tau_* v_*)_{x=\pm 1} = (\tau_* u_*)_{y=\pm 1} = 0, \\
p_*,\ q_*,\ \frac{\partial p_*}{\partial x},\ \frac{\partial q_*}{\partial y} \in L_2(\Omega), \\
\tau_*,\ u_*,\ v_* \in W_2^1(\Omega).
\end{gathered} \tag{3.3}$$

It is assumed that p, q, τ, u, v have quadratically summable, with respect to Ω, derivatives and generalized sums of derivatives which enter into (3.1) and (3.2); one of the factors in (3.3) [the same as in (1.1)] is zero over the entire side of the square.

From the sequence of the solutions (2.1) we can extract a subsection which weakly converges in $L_2(\Omega)$ to the generalized solution. If there exists a generalized solution which satisfies the conditions (3.3), then for n, m → ∞ the entire sequence of solutions (2.1) converges to it. These assertions are proved in the same way as the analogous assertions in [1].

4. Reduction of the Problem to a Sequence of Boundary-Value Problems for Ordinary Equations

We denote

$$p_n=\sum_{k=0}^{n} p_k^n P_k,\quad q_n=\sum_{k=0}^{n} q_k^n P_k,\quad \tau'_n=\sum_{k=0}^{n-1}\tau_k^n P_k,$$
$$\tau''_n=\sum_{k=0}^{n+1}\tau_k^n P_k,\quad u'_n=\sum_{k=0}^{n} u_k^n P_k,\quad u''_n=\sum_{k=0}^{n+2} u_k^n P_k,\tag{4.1}$$
$$v'_n=\sum_{k=0}^{n+1} v_k^n P_k,\quad v''_n=\sum_{k=0}^{n-1} v_k^n P_k,\quad t_n=\sum_{k=0}^{n-1} t_k^n P_k,$$

where p_k^n, q_k^n, τ_k^n, u_k^n, v_k^n, t_k^n are functions of x; $P_k=P_k(y)$ are Legendre polynomials; and k is the degree of the polynomial.

The approximate solution of the mixed problem for a hollow elastic cylinder is sought in the form of the functions (4.1) which satisfy the equations

$$\int_{-1}^{1}\left(\frac{\partial p_n}{\partial x}+\frac{\partial \tau''_n}{\partial y}+f_1\right)P_k dy=0,$$
$$\int_{-1}^{1}\left[p_n-r\left(\lambda\varepsilon_n+2\mu\frac{\partial u'_n}{\partial x}\right)+f_3\right]P_k dy=0,$$
$$\int_{-1}^{1}\left[q_n-r\left(\lambda\varepsilon_n+2\mu\frac{\partial v'_n}{\partial y}\right)+f_4\right]P_k dy=0,\quad k=0,1,\ldots,n;$$
$$\int_{-1}^{1}\left[\tau''_n-\mu r\left(\frac{\partial u''_n}{\partial y}+\frac{\partial v''_n}{\partial x}\right)+f_5\right]P_k dy=0,\quad k=0,1,\ldots,n+1;\tag{4.2}$$
$$\int_{-1}^{1}\left(\frac{\partial \tau'_n}{\partial x}+\frac{\partial q_n}{\partial y}-t_n+f_2\right)P_k dy=0,$$
$$\int_{-1}^{1}\left(t_n-\lambda\varepsilon_n-2\mu\frac{v''_n}{r}\right)P_k dy=0,$$
$$\int_{-1}^{1}\left[\tau'_n-\mu r\left(\frac{\partial u''_n}{\partial y}+\frac{\partial v''_n}{\partial x}\right)+f_5\right]dy=0,\quad k=0,1,\ldots,n-1,$$
$$\varepsilon_n=\frac{\partial u'_n}{\partial x}+\frac{\partial v'_n}{\partial y}+\frac{v''_n}{r}$$

and the boundary conditions

$$(q_n v'_n)_{y=\pm 1}=(\tau''_n u''_n)_{y=\pm 1}=0;\tag{4.3}$$

$$(p_n^n u_n^n)_{x=\pm 1}=(p_k^n u_k^n)_{x=\pm 1}=(\tau_k^n v_k^n)_{x=\pm 1}=0,\tag{4.4}$$
$$k=0,\ 1,\ \ldots,\ n-1.$$

It is assumed that in each of Eqs. (4.3) one of the factors [the same as in (1.1)] is zero over the entire side of the square.

If the formulation of the problem contains Eq. (1.2), then the system (4.2), (4.3) is supplemented by the equation

$$\int_{-1}^{1} u_0^n dx=0.$$

We can introduce the concept of a generalized solution of the boundary-value problem for equations (4.2), (4.3) with the boundary conditions (4.4), analogously to the corresponding concept in [1]. The proofs of the existence and uniqueness of this generalized solution, its convergence to the generalized solution to the mixed problem for a hollow elastic cylinder and are analogous to the corresponding proofs in [1].

LITERATURE CITED

1. G. V. Ivanov, "Solution of a plane mixed problem of the theory of elasticity in the form of series of Legendre polynomials," Zh. Prikl. Mekh. Tekh. Fiz., No. 6 (1976).

PROBLEM OF PLANE STRAIN OF HARDENING AND SOFTENING PLASTIC MATERIALS

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§1. The classical description of the kinematics of deformation of a solid medium is based on the assumption of sufficient smoothness of the displacement field. The smoothness assumption allows us to introduce the concept of a strain tensor and make use of the tool of differential equations for the description of the deformation of the material. However, there exist broad classes of motion of the medium where the displacement can be connected with the appearance of a plastic strain. Experiments with various materials show that the mechanism of plastic strain is connected with localization of shear along certain surfaces [1, 2]. The latter signifies that on these certain surfaces the displacement vector experiences a violent break. In the general case this circumstance turns out to be important and must be taken into account when describing plastic deformation. Making certain assumptions which are justified from a mechanical viewpoint, we can describe a nonsmooth displacement field by fairly simple means with the aid of a collection of smooth functions.

As an illustration, we shall consider the case of a single function of a single variable. By F(x) we denote the original function having a violent break at the points x_i. We assume that the distances between the breaks are small, that the function F(x) is sufficiently smooth between the break points, and that the values of the derivatives of F(x) on the right and on the left of the break points are equal to one another; i.e., the function

$$p(x)=\begin{cases} F'(x) & \text{for} \quad x\neq x_i,\\ F'(x_i\pm 0) & \text{for} \quad x=x_i\end{cases}$$

is sufficiently smooth.

Let f(x) be a smooth function satisfying the conditions $f(x_i)=F(x_i+0)$ and $P(x)=\int p(x)\,dx$. Then the original function F(x) can be characterized by a pair of smooth functions f(x), P(x) and a sequence of break points x_i (Fig. 1). The function f(x) has the meaning of averaging the original function, and it characterizes (with a certain accuracy) the values of F(x) over the entire domain of definition. The function $P'(x)-f'(x)$ characterizes the difference in local behavior of the original and the averaged functions, and for given break points determines the magnitude of jumps of the original function. Thus, a jump of the function F(x) at the point x_{i+1} with an accuracy up to l_i^2 equals $\{f'(x_i)-P'(x_i)\}l_i$ where $l_i=x_{i+1}-x_i$ is the distance between the adjacent break points.

Analogously, we shall consider the case of a vector function $\mathbf{V}=V_1\mathbf{e}_1+V_2\mathbf{e}_2$ of the vector argument $\mathbf{r}=x_1\mathbf{e}_1+x_2\mathbf{e}_2$ ($\mathbf{e}_1$, $\mathbf{e}_2$ is the orthonormed basis). Let $l\ll 1$ be a characteristic dimension of the regions where the function $\mathbf{V}(\mathbf{r})$ is sufficiently smooth. We shall call such regions elements. We assume that for the original functions there exists a smooth average $\mathbf{v}(\mathbf{r})$ such that $\mathbf{v}(\mathbf{r}_i)=\mathbf{V}(\mathbf{r}_i)$ at the centers of elements $\mathbf{r}_i$; on the boundary separating elements with the centers at the points $\mathbf{r}_i$, $\mathbf{r}_{i+1}$ the break of $\mathbf{V}(\mathbf{r})$ with an accuracy up to $|\mathbf{r}_{i+1}-\mathbf{r}_i|^2$ equals $A(\mathbf{r}_i)(\mathbf{r}_{i+1}-\mathbf{r}_i)$, where A is a tensor of the second rank with smooth components A_{km}, k, m = 1, 2. A smooth vector field $\mathbf{v}(\mathbf{r})$ and a tensor field $A(\mathbf{r})$ are put in correspondence with the original field $\mathbf{V}(\mathbf{r})$. It can be shown that the description thus introduced imposes the following constraint on the class of discontinuous functions: On the break lines the values of one-sided derivatives $\partial V_k/\partial x_m$ must be equal to one another. It is obvious that

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